

# UNCLASSIFIED

AD NUMBER
AD264132
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational use; Sep 1961. Other requests shall be referred to Office Of Naval Reseaearch, Arlington VA.
AUTHORITY
Onr ltr, 28 Jul 1977

THIS PAGE IS UNCLASSIFIED

THIS REPORT HAS BEEN DELIMITED  
AND CLEARED FOR PUBLIC RELEASE  
UNDER DOD DIRECTIVE 5200.20 AND  
NO RESTRICTIONS ARE IMPOSED UPON  
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE;  
DISTRIBUTION UNLIMITED.

UNCLASSIFIED

---

AD 264 132

*Reproduced  
by the*

ARMED SERVICES TECHNICAL INFORMATION AGENCY  
ARLINGTON HALL STATION  
ARLINGTON 12, VIRGINIA



---

UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

NOX

MICHIGAN STATE UNIVERSITY  
PHYSICS DEPARTMENT  
ULTRASONICS LABORATORY

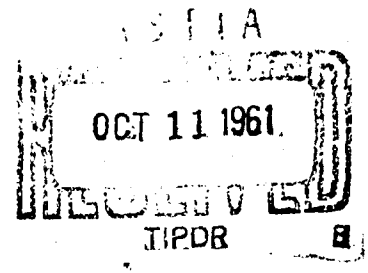
FOR REFERENCE ONLY AT EACH OF THE  
ASTIA OFFICES. THIS REPORT CANNOT  
BE SATISFACTORILY REPRODUCED; ASTIA  
DOES NOT FURNISH COPIES.

Office of Naval Research  
Contract Nonr-2587(01)  
Project No. NR 385-425

OPTICAL METHODS FOR ABSOLUTE MEASUREMENT OF  
SOUND PRESSURE IN LIQUIDS.

Technical Report No. 4.

E. A. Hiedemann  
Main Investigator  
September 1961



Reproduced From  
Best Available Copy

AD NO. —  
ASTIA FILE COPY

264132

This Technical Report consists of papers published between January and July, 1961 by the staff and graduate students supported by the Office of Naval Research.

Staff:

E. A. Hiedemann	Professor of Physics (Research)
M. A. Breazeale	Assistant Professor (Research)
W. G. Mayer	Assistant Professor (Research)

Graduate Students:

W. W. Lester  
W. R. Klein

TABLE OF CONTENTS

- I. M. A. Broezeale and E. A. Hiedemann, "Diffraction Patterns Produced by Finite Amplitude Waves". J.A.S.A. 33, 700 (1961).
- II. W. G. Mayer and E. A. Hiedemann, "Light Diffraction by Progressive Ultrasonic Waves in Plexiglas". Acustica 10, 251 (1960. (Manuscript of this paper was included in Technical Report No. 3)).
- III. B. D. Cook and E. A. Hiedemann, "Diffraction of Light by Ultrasonic Waves of Various Standing Wave Ratios". J.A.S.A. 33, 945 (1961).
- IV. M. A. Broezeale, "Experimental Studies of 'Least Stable' Waveform". Presented at the 61st National Meeting of the A.S.A., May 1961.
- V. Distribution List.

## Diffraction Patterns Produced by Finite Amplitude Waves

M. A. BREAZEALE AND E. A. HIEDEMANN  
 Department of Physics, Michigan State University,  
 East Lansing, Michigan

(Received February 6, 1961)

Pictures are given which illustrate in detail the optical effects produced by ultrasonic waves of finite amplitude in liquids. These effects can be used for the analysis of the wave form.

THE theory of the diffraction of light by an ultrasonic wave given by Raman and Nath<sup>1</sup> has been extended by Zankel and Hiedemann<sup>2</sup> to the diffraction by distorted finite amplitude waves. They give both theoretical and experimental results in the form of plots of the intensity of the first-, second-, and third-diffraction orders as a function of the Raman-Nath parameter  $v = 2\pi\mu L/\lambda$  for various distances. From these curves one can see that the intensities of the diffracted orders pass through maxima and minima, going to zero at certain  $v$  values. The way in which the intensities of the various orders change with sound intensity and wave form can be seen even more readily from photographs of the diffraction patterns. Photographs showing the intensity distribution in the diffraction orders at different ultrasonic intensities for sinusoidal waves have been given by Nomoto.<sup>3</sup> Photographs of the images produced by finite amplitude waves, using both wide and narrow light beams, will be presented here.

Since the distortion in a finite amplitude wave increases with

both ultrasonic intensity and distance from a sinusoidally vibrating transducer, the asymmetry of a diffraction pattern, formed when collimated light passes through the wave, increases both with ultrasonic intensity and with distance from the source. This can be seen in Fig. 1. The distances from a 1.76-Mc 6-cm diam quartz transducer are given at the top. The values  $V$  given at the left are values of the Raman-Nath parameter  $v = 2\pi\mu L/\lambda$ . For this transducer the value  $v = 7.5$  corresponds roughly to a pressure amplitude at the transducer of 1 atm.

The pictures in the first column were made for those values of pressure amplitude which gave alternately zeros of intensity in the zero- and first-diffraction orders. The pictures in the second and third column were made at the same initial pressure amplitudes. It can be seen that because of distortion and of absorption of the wave the zeros of intensity are no longer necessarily in the same orders. The zeros of the diffracted orders are asymmetrical, as are the intensity maxima. The positions of the maxima at the edges of

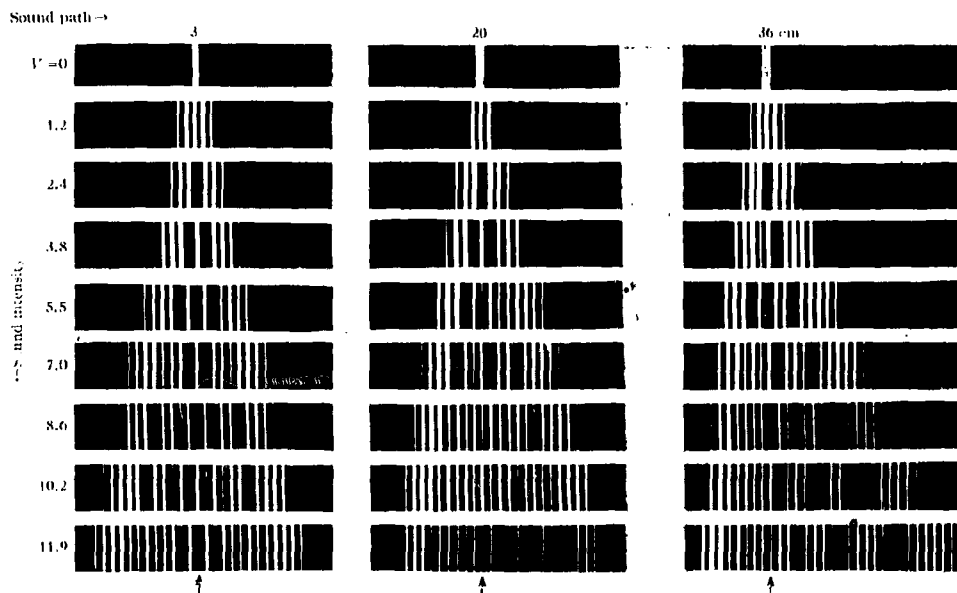


FIG. 1. Light diffraction by ultrasonic waves in water.  $f = 1.76$  Mc.



FIG. 2. "Narrow beam diffraction" by ultrasonic waves in water.  $F=1.76$  Mc.  $W=\lambda/2$ .

the pattern are also determined by the wave form. Mikhailov and Shutilov<sup>4</sup> have developed a method for inferring the ultrasonic wave form from the positions of these maxima.

Pictures of the diffraction patterns produced by distorted ultrasonic waves have been given by Mikhailov and Shutilov.<sup>4-6</sup> The way the zeros of light intensity vary with ultrasonic intensity is not obvious in their photographs because of the high ultrasonic intensities used. Minima of intensity are visible in Fig. 3 of Shutilov.<sup>7</sup>

A homogeneous portion of the ultrasonic beam was used in these experiments in order to secure zeros of intensity in the diffraction pattern. This was accomplished by making the cross section of the light beam  $\sim 3$  mm in diam.

If the light beam width is less than one ultrasonic wave length, discrete diffraction patterns are no longer formed. The image appears broadened. If the ultrasonic wave is distorted the broadened image is asymmetrical, the more intense side of the image being closer to the undeflected position and on the side of the optical axis opposite the ultrasonic source<sup>8</sup>; i.e., the direction of propagation of ultrasound is from left to right in both Figs. 1 and 2. Broadened images are shown in Fig. 2 for conditions identical with those for Fig. 1, with a light beam width equal to half the

ultrasonic wavelength. The theory of narrow beam diffraction and its experimental verification have been given by Hargrove, Zankel, and Hiedemann.<sup>9</sup> The width of the patterns is the same as that of the corresponding diffraction patterns in Fig. 1. These images can be thought of as "smeared out" images of the discrete diffraction patterns, although the light intensity does not go to zero within the pattern. To secure such zeros without discrete orders, the light beam width should be between one-half and one ultrasonic wavelength. Such patterns, produced by sinusoidal waves are given by Breazeale and Hiedemann.<sup>10</sup>

<sup>1</sup> C. V. Raman and N. S. N. Nath, *Proc. Indian Acad. Sci.* **2**, 406 (1935); **2**, 413 (1935); **3**, 75 (1936); **3**, 119 (1936).

<sup>2</sup> K. L. Zankel and E. A. Hiedemann, *J. Acoust. Soc. Am.* **31**, 44 (1959).

<sup>3</sup> O. Nomoto, *Proc. Phys. Math. Soc. Japan* [3] **22**, 314 (1940).

<sup>4</sup> I. G. Mikhailov and V. A. Shutilov, *Soviet Phys.—Acoustics* **4**, 174 (1958).

<sup>5</sup> I. G. Mikhailov and V. A. Shutilov, *Soviet Phys.—Acoustics* **3**, 217 (1957).

<sup>6</sup> I. G. Mikhailov and V. A. Shutilov, *Soviet Phys.—Acoustics* **5**, 75 (1959).

<sup>7</sup> V. A. Shutilov, *Soviet Phys.—Acoustics*, **5**, 230 (1959).

<sup>8</sup> M. A. Breazeale and E. A. Hiedemann, *J. Acoust. Soc. Am.* **30**, 751 (1958).

<sup>9</sup> J. Hargrove, K. L. Zankel, and E. A. Hiedemann, *J. Acoust. Soc. Am.* **31**, 1366 (1959).

<sup>10</sup> M. A. Breazeale and E. A. Hiedemann, *J. Acoust. Soc. Am.* **31**, 24 (1959).

## LIGHT DIFFRACTION BY PROGRESSIVE ULTRASONIC WAVES IN PLEXIGLAS\*

by W. G. MAYER and E. A. HIEDEMANN

Physics Department Michigan State University, East Lansing, Michigan, U.S.A.

## Summary

The optical grating produced by a progressive ultrasonic wave in Plexiglas is used to verify directly the Raman-Nath theory for solids after it is shown that no finite amplitude distortion of the waveform is present. The absorption coefficient is determined by evaluating the diffraction pattern.

## Zusammenfassung

Das von einer fortschreitenden Ultraschallwelle in Plexiglas erzeugte optische Gitter wird zum direkten Beweis der Raman-Nathschen Theorie für Festkörper benutzt, nachdem gezeigt wird, daß keine Aufsteilung der Wellenform durch endliche Amplituden festzustellen ist. Der Absorptionskoeffizient wird durch Auswertung von Beugungsspektren bestimmt.

## Sommaire

Pour vérifier directement la théorie de Raman-Nath sur les solides, on utilise le réseau optique produit par une onde ultrasonore progressive dans le plexiglas, après avoir démontré qu'aucune distortion finie de la forme d'onde n'est présente. Le coefficient d'absorption est déterminé par l'évaluation des spectres de diffraction.

## 1. Introduction

The theory of RAMAN and NATH [1] predicts that the light intensity in the  $n^{\text{th}}$  order of a diffraction pattern produced by a progressive ultrasonic wave is given by

$$I_n = J_n^2(v), \quad (1)$$

where  $J_n$  is the  $n^{\text{th}}$  order Bessel function of argument  $v$ . The parameter  $v$  is given by

$$v = 2\pi\mu L/\lambda. \quad (2)$$

Here  $L$  is the path length of the light in the ultrasonic beam,  $\mu$  the maximum change of index of refraction, and  $\lambda$  the wavelength of the light.

SANDERS [2], NOMOTO [3] and others have found satisfactory agreement between the results of their experimental work in liquids and eq. (1). However, there are some minor discrepancies between the measured values and the theoretical curves. Some of these deviations may be explained in the light of recent experiments by ZAREMBO, KRASILNIKOV and SHKLOVSKAYA-KORDI [4] and ZANKEL and HIEDEMANN [5] who showed that the presence of higher harmonic components in an originally sinusoidal ultrasonic wave will not only cause asymmetries in the light distribution of the diffraction pattern but will also shift the maxima and minima of a diffraction order to other values of  $v$  than predicted by eq. (1). Since finite amplitude distortions (second harmonics) are commonly present in the ranges where measurements are normally made, the slight dis-

agreement between theory and measured values in some liquids may be attributed to this effect. Furthermore, using a liquid with moderate absorption, it is more difficult to produce a pure progressive wave than was realized in some early investigations. Reflected waves will tend to influence the light intensity distribution of the diffraction pattern.

## 2. Procedure

The RAMAN-NATH theory was verified indirectly for solids by the work of MUELLER [6], [7] dealing with elasto-optical constants. Direct verifications of the light intensity distribution in the diffraction pattern were reported only for liquids. In solids like glasses and transparent crystals only the effects of stationary waves were observed on account of the small sound absorption in these substances. PROTZMAN [8] used the spacing of the diffraction pattern produced by ultrasonic waves to measure the sound velocity in Plexiglas where the absorption is very high as reported by MASON and McSKIMIN [9]. Plexiglas should therefore be useful for the study of the diffraction pattern produced by a progressive wave in a solid; the high absorption coefficient makes it possible to exclude reflected waves. In the experiment described here a block of Plexiglas is used which has the dimensions 10 cm  $\times$  20 cm  $\times$  60 cm. Considering the value of absorption given by MASON and McSKIMIN,  $\alpha = 5$  dB/cm for 2.5 Mc/s, one sees that an ultrasonic wave of that frequency travelling in the direction of the longest dimension of the block will return to the transducer attenuated by several hundred dB.

\* This work was supported by the Office of Naval Research, U.S. Navy, and by the National Science Foundation, Grant NSF-G 6318.

## Diffraction of Light by Ultrasonic Waves of Various Standing Wave Ratios\*

B. D. COOK† and F. A. HIEDEMANN  
 Department of Physics, Michigan State University, East Lansing, Michigan  
 (Received January 16, 1961)

The theory for the diffraction of light by plane ultrasonic waves of various standing wave ratios is derived. The liquid medium disturbed by the ultrasound is considered to act as an optical phase grating. By evaluating the diffraction integral for the light amplitude, expressions for the Doppler shift, the time dependent, and the time average light intensities are found for the diffraction spectrum. Experimental measurements using two adjacent ultrasonic waves progressing in opposite directions to simulate the desired optical phase grating indicate that the theory is valid.

### INTRODUCTION

THE previous studies of the diffraction of light by ultrasound have considered two extreme cases: the progressive wave and the stationary wave. In this investigation the simplified Raman and Nath theory is extended to the diffraction of light by ultrasound of various standing wave ratios.

The simplified theory of Raman and Nath<sup>1</sup> considers the medium to act as an optical phase grating and assumes that there is no amplitude modulation of the light wavefront emerging from the ultrasound. For progressive and stationary waves this theory is successful in predicting not only the separation and intensities of the diffracted orders but also the Doppler shift of the diffracted light. Other theoretical considerations have shown that the simplified theory is valid over a wide range of experimental conditions.

For both progressive and stationary waves Sanders<sup>2</sup> experimentally confirmed the light intensities given by the Raman and Nath theory. Debye, Sack, and Coulon<sup>3</sup> first experimentally detected the Doppler shift. By using magnetic detuning of the mercury resonance line, Ali<sup>4</sup> has directly measured the Doppler shift for progressive waves.

The light intensity in the diffracted orders is also time dependent for standing waves of finite standing wave ratio (SWR). These orders can be used as a light source for a stroboscope. Therefore, to fully understand the principles of the ultrasonic modulation stroboscope, it is necessary to understand how light is diffracted by ultrasonic waves of various SWR.

A two-transducer method is used to produce an optical grating corresponding to one produced by a stationary wave. The light is diffracted successively by two adjacent, oppositely directed sound beams. Others<sup>5-7</sup> have used this method to some success.

\* This research was supported by the Office of Ordnance Research, U. S. Army, and by the Office of Naval Research.

† National Science Foundation Cooperative Fellow.

<sup>1</sup> C. V. Raman and N. S. Nath, Proc. Indian Acad. Sci. A2, 406 (1935); 3, 75 (1936).

<sup>2</sup> F. H. Sanders, Can. J. Research (A) 14, 158 (1936).

<sup>3</sup> P. Debye, H. Sack, and F. Coulon, Compt. rend. 198, 922 (1934).

<sup>4</sup> L. Ali, Helv. Phys. Acta 9, 63 (1936).

<sup>5</sup> R. Bär, Helv. Phys. Acta, 9, 678 (1936).

<sup>6</sup> A. Pande, M. Pancholy, and S. Parthasarathy, J. Sci. Ind. Research 3, 64 (1944).

<sup>7</sup> A. Giacomini, Ricerca Sci. 18, 803 (1948).

Recently, Mertens<sup>8</sup> has theoretically given the criteria for the conditions under which successive diffraction by two sound beams is the same as simultaneous diffraction.

### THEORY

The simplified Raman and Nath theory evaluates a diffraction integral for a phase grating. For a collimated monochromatic light beam of width  $2D$  at normal incidence to an infinite transmission phase grating, the amplitude distribution of the diffracted light is given by

$$A(\theta) = Ce^{i\omega t} \int_{-D}^D \exp\{i[ux + v(x,t)]\} dx, \quad (1)$$

where  $u = 2\pi/\lambda$  and  $l = \sin\theta$ .  $\lambda$  is the wavelength of the light,  $\omega$  is the angular frequency of the light, and  $\theta$  is the angle at which the light is diffracted;  $C$  is a normalization constant. The term  $v(x,t)$  is a dimensionless parameter describing the optical phase grating in space and time.

This parameter  $v(x,t)$  is related to the instantaneous sound pressure through the change of index of refraction. For plane ultrasonic waves of instantaneous pressure  $p(x,t)$ ,

$$v(x,t) = (2\pi L\kappa/\lambda)p(x,t), \quad (2)$$

where  $\kappa$  is the piezo-optic constant for the medium and  $L$  is the width of the sound beam.

The instantaneous pressure in a sinusoidal standing wave may be written as

$$p(x,t) = p \sin(\omega^*t - k^*x) + (p/a) \sin(\omega^*t + k^*x), \quad (3)$$

where  $\omega^*$  and  $k^*$  are the acoustic angular frequency and wave constant, respectively. The quantity  $a$  is the standing wave ratio (SWR), i.e., the ratio of the amplitude of the incident wave to the amplitude of the reflected wave. For such a standing wave the diffraction integral becomes

$$A(\theta) = Ce^{i\omega t} \int_{-D}^D \exp\left\{i\left[ux + p \sin(\omega^*t - k^*x) + \left(\frac{r}{a}\right) \sin(\omega^*t + k^*x)\right]\right\} dx, \quad (4)$$

<sup>8</sup> R. Mertens, Z. Physik 160, 291 (1969).

Using the identity

$$e^{ib \sin t} = \sum_{r=-\infty}^{\infty} J_r(b) e^{ir t}, \quad (5)$$

where  $J_r$  is the  $r$ th order Bessel Function, Eq. (4) upon integration gives

$$A(\theta) = 2C e^{i\omega t} \left( \sum_s \sum_r J_r(v) J_s\left(\frac{v}{a}\right) \exp[i(r+s)\omega^* t] \right. \\ \left. \times \{\sin[ul + (s-r)k^*] D\} / [ul + (s-r)k^*] \right). \quad (6)$$

To normalize, it is assumed that the light amplitude at  $\theta=0$  equals unity for  $v=0$ ; thus one finds  $C=1/2D$ . Let  $n=r-s$ , and then Eq. (6) becomes

$$A(\theta) = \sum_n \sum_r J_r(v) J_{r-n}\left(\frac{v}{a}\right) W_n \\ \times \exp\{i[(2r-n)\omega^* + \omega]t\}, \quad (7)$$

where

$$W_n = \sin(nl - nk^*) D / (nl - nk^*) D. \quad (8)$$

For ideal diffraction,  $D \rightarrow \infty$ .  $W_n$  then has nonzero values only for

$$nl - nk^* = 0, \quad (9)$$

which means that light occurs only at angles given by

$$\sin \theta = n\lambda / \lambda^*. \quad (10)$$

Thus the light is diffracted into discrete orders of amplitude  $A_n$  given by

$$A_n = \sum_{r=-\infty}^{\infty} J_r(v) J_{r-n}\left(\frac{v}{a}\right) \exp\{i[(2r-n)\omega^* + \omega]t\}. \quad (11)$$

The interpretation given to Eq. (11) is that the light in the orders has undergone different amounts of Doppler shift. For ultrasound of frequency  $\nu^*$  and

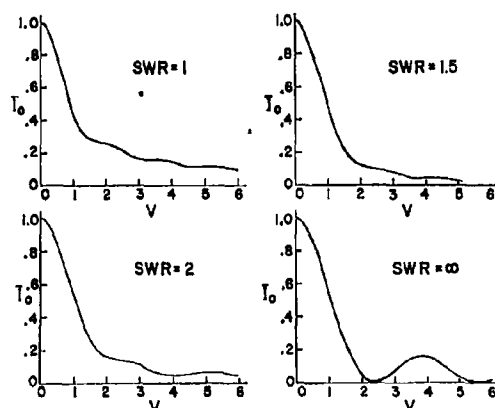


FIG. 1. Theoretical average light intensities in the zero orders for different SWR.

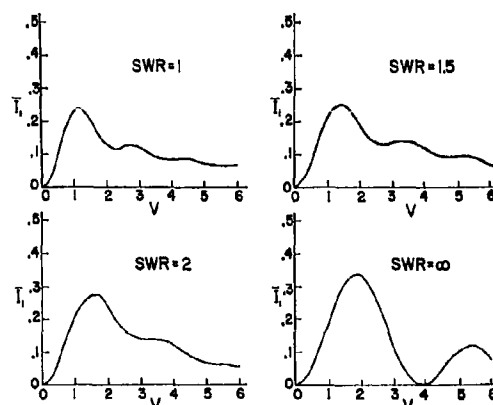


FIG. 2. Theoretical average light intensities in the first orders for different SWR.

incident light of frequency  $\nu$ , the  $n$ th order contains light of frequencies  $\nu + (2r-n)\nu^*$  with amplitudes  $J_r(v) J_{r-n}(v/a)$ , where  $r$  takes on all integral values, positive and negative. The amplitude of these non-coherent components of the light in each order depends on the SWR while the actual frequency shift does not.

The time-dependent intensity in the  $n$ th order is

$$I_n = A_n A_n^* \\ = \sum_{p=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} J_r(v) J_{r-n}\left(\frac{v}{a}\right) J_p(v) J_{p-n}\left(\frac{v}{a}\right) \\ \times \exp[i2(r-p)\omega^* t] \quad (12)$$

and the time average light intensity is

$$\bar{I}_n = \sum_{r=-\infty}^{\infty} J_r^2(v) J_{r-n}^2\left(\frac{v}{a}\right). \quad (13)$$

Equation (13) reduces to the Raman and Nath results for both progressive and stationary waves. For progressive waves ( $a \rightarrow \infty$ ), the light in the  $n$ th order is of a single frequency  $\nu + n\nu^*$ . The expressions for the time-dependent and average light intensities are identical. They are given by

$$I_n = \bar{I}_n = J_n^2(v) \quad (14)$$

as

$$J_{r-n}(0) = \delta_{rn}. \quad (15)$$

For stationary waves the time-dependent light intensity is expressed in a more convenient form if one writes for the phase grating

$$v(x, t) = 2v \cos \omega^* t \sin k^* x. \quad (16)$$

After integrating and normalizing as above, the time-dependent light intensity becomes

$$I_n = J_n^2(2v \cos \omega^* t). \quad (17)$$

From Eq. (13) or by taking the time average of Eq. (17) one finds that the time average light intensity for a stationary wave is

$$\bar{I}_n = \sum_{r=-\infty}^{\infty} J_r^2(v) J_{r-n}^2(v). \quad (18)$$

The time dependence of Eq. (17) follows from Eq. (14) if one considers the grating model. For a progressive wave the grating is translating in its own plane. Translation of an infinite grating does not change the light distribution in the diffracted orders. For a stationary wave one may consider the grating fixed in space and the magnitude of the phase variation to be sinusoidal in time. Thus the light intensities of the orders vary in time as the magnitude of the phase grating varies.

Equations (17) and (18) are the same as those given by Raman and Nath<sup>1,9</sup> except that the equations of Raman and Nath have  $\frac{1}{2}v$  rather than  $v$  as the

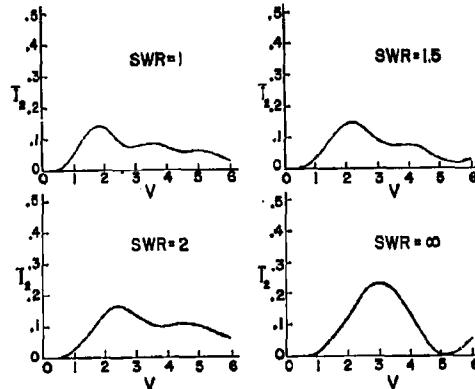


FIG. 3. Theoretical average light intensities in the second orders for different SWR.

argument of the Bessel functions. This difference arises because the Raman and Nath  $v$  refers to the maximum amplitude in the stationary wave, whereas, the  $v$  in Eqs. (17) and (18) refers to the amplitude of the separate opposite-directed waves.

Figures 1-4 show theoretical curves for the average light intensities in the zero, first, second, and third orders, respectively, for various standing wave ratios. Only for progressive waves do the minimums of the light intensities become zeros.

#### EXPERIMENTAL ARRANGEMENT

An optical phase grating corresponding to that of an ultrasonic standing wave is produced with adjacent progressive waves traveling in opposite directions. The optical axis intercepts both sound beams normally. The arrangement of two adjacent sound beams is used because in this way the SWR can be readily

<sup>9</sup> N. S. Nath, *Akust. Z.* 4, 263 (1939).

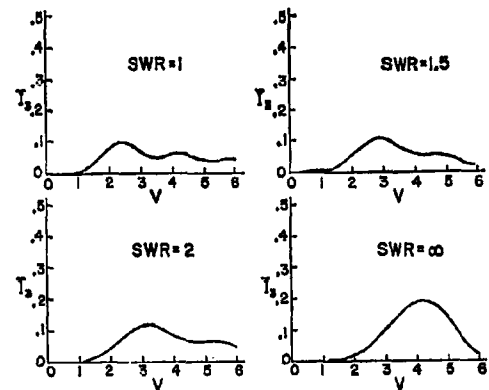


FIG. 4. Theoretical average light intensities in the third orders for different SWR.

determined by optical means. For a certain applied rf potential to the transducers, the  $v$  value of each sound may be determined individually from Eq. (14) if the other transducer is placed such that its sound beam does not cross the optical path. The two progressive waves are obtained by eliminating reflections in a specially designed tank. This tank is a modification on the one described by Hargrove, Zankel, and Hiedemann.<sup>10</sup> The sound traveling in each direction is absorbed by a castor oil termination.

A schematic diagram is shown in Fig. 5. The mercury light source illuminates the slit SL. The collimated beam produced by lens  $L_2$  is normal to both sound beams. The light intensities of the diffracted orders are measured by a photomultiplier.

Two air-backed quartz transducers are driven by a crystal-controlled 500-w transmitter. The transducers are 1 in. sq. The relative ultrasonic output of the transducer is controlled with a variable inductance used as an impedance match between transmitter and the transducers.

The average light intensity is measured with a microphotometer photomultiplier. For time-dependent measurements the output signal of the photomultiplier

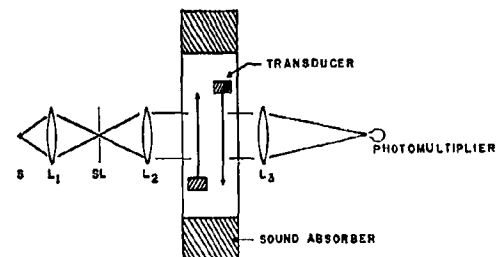


FIG. 5. Schematic diagram of experimental apparatus.

<sup>10</sup> L. E. Hargrove, K. L. Zankel, and E. A. Hiedemann, *J. Acoust. Soc. Am.* 31, 1366 (1959).

is amplified by two wide-band amplifiers in cascade and displayed on an oscilloscope.

To avoid finite amplitude effects the measurements are made at low intensities and close to the transducers. All measurements are made in water at room temperature at the frequency of 1 Mc.

#### EXPERIMENTAL OBSERVATIONS

Figure 6 shows experimental values and theoretical curves for three orders at  $\text{SWR}=2$ . Within experimental accuracy the orders are symmetric about the zero order. The agreement between theory and experiment is best at small  $v$  values. At higher  $v$  values the deviations are probably caused by the fact that the theory assumes that the light is diffracted simultaneously by the two beams while the experimental arrangement is such that it is diffracted first by one beam, then by the other. Using the reasoning of Mertens,<sup>8</sup> one would expect higher discrepancies at higher  $v$  values for successive diffraction as the light may be bent significantly by the first beam.

The shape of oscilloscope traces representing the time-dependent intensities are similar to the curves predicted by Eqs. (12) and (17). Maximum modulation is observed to occur when  $\text{SWR}=1$ .

#### MODULATION STROBOSCOPES

As the maximum time-dependent modulation occurs when  $\text{SWR}=1$ , the diffracted orders caused by the stationary waves are used extensively as sources of modulated light especially in stroboscopes. The zero order provides two pulses of light per period of ultrasound. This pulse width is dependent on the parameter  $v$  as shown in Fig. 7. In general the pulse width decreases as the parameter  $v$  is increased; however, satellite pulses occur at  $v$  greater than three.

Figure 7 also shows the time-dependent light intensity

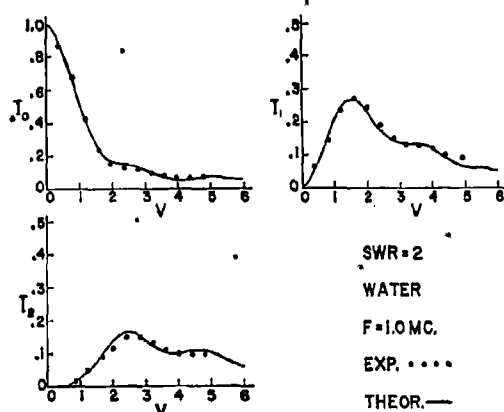


Fig. 6. Experimental and theoretical light intensities of the zero, first, and second orders for  $\text{SWR}=2$ .

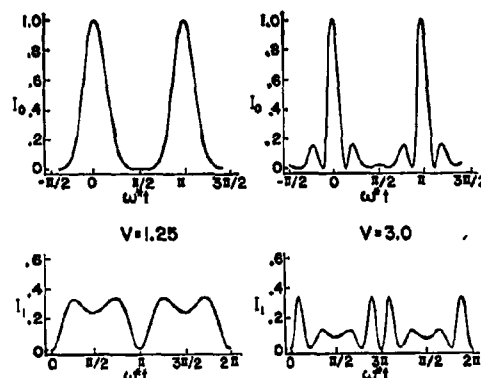


Fig. 7. Theoretical time dependent light intensities of the zero and first orders for a stationary wave of  $v=1.25$  and  $v=3.0$ .

for the first order for two values of  $v$ . It is evident from Eq. (17) that the orders higher than the first will also exhibit time-dependence similar to that of the first order. The modulation of these less-intense higher orders is not as suitable for stroboscopic use as the zero order.

Equation (12) gives the harmonic structure of the magnitude of the light intensities of the diffracted orders for any  $\text{SWR}$ . From this equation it is seen that the fundamental frequency of light modulation is  $2v^*$  for all finite  $\text{SWR}$ . For all finite  $\text{SWR}<1$ , the diffracted orders are not completely modulated but each has residual light which does not undergo modulation.

#### SUMMARY

Expressions for the diffraction of light by plane ultrasonic waves of various standing wave ratios are derived using the method of Raman and Nath. This method consists of evaluating a diffraction integral for an optical phase grating. The expressions for the light intensity of the diffraction spectrum reduce to those given by Raman and Nath for progressive and stationary waves.

The light intensities in the diffracted orders are found to be time dependent for standing waves with finite standing wave ratio ( $\text{SWR}$ ). In application to diffraction type stroboscopes, the amount of light modulation in the diffracted orders can now be determined for various  $\text{SWR}$ .

It is theoretically shown that the magnitudes of noncoherent components resulting from Doppler shift depend on the  $\text{SWR}$ . Further, for finite  $\text{SWR}$ , the frequency shift is found to be independent of  $\text{SWR}$ .

Measurements of time-dependent and average light intensities indicate that the theoretical results are valid.

#### ACKNOWLEDGMENT

The authors wish to thank Dr. Kenneth L. Zankel for valuable discussions of the problem.

# EXPERIMENTAL STUDIES OF THE "LEAST STABLE WAVEFORM"

On propagating through a liquid, an ultrasonic wave of large amplitude can undergo distortion so that the slope of its leading edge is much greater than that of its trailing edge. If such a non-symmetric wave is reflected from a boundary, two extreme cases are possible: the waveform may be unchanged, as in the case of a perfectly rigid reflector, or it may be inverted, as in the case of a perfect pressure release reflector. For a pressure release reflector, the resulting waveform would have the slope of its trailing edge greater than the slope of its leading edge. Such a waveform has been described by Fay<sup>1</sup> as the "least stable waveform" in contrast with the more usual case of the most stable waveform. In the least stable waveform the fundamental harmonic component can increase with distance, while the higher harmonics decrease with distance<sup>2</sup>. Thus, an initially distorted wave can become undistorted as it progresses. The following experiment was designed to demonstrate the inversion of waveform on reflection from a pressure release boundary and this decrease of distortion with distance.

A pressure release reflector was constructed by stretching a plastic membrane over an air chamber and a rigid one by mounting an aluminum plate so it could be easily attached in place of the pressure release boundary.

The experiment was performed with high intensity 2 Mc ultrasonic pulses in water. The waveform of the ultrasonic pulse at various distances before and after it had been reflected was monitored by use of a barium titanate transducer. Since the receiver resonated near the second harmonic, this harmonic was accentuated in the waveform displayed on an oscilloscope. Further, the receiver resonance caused a phase shift of almost  $90^\circ$  in the second harmonic component displayed on the oscilloscope. This made it possible to detect very small amounts of second harmonic by observing the "flattening" of the bottom of the oscilloscope trace and the "sharpening" of the top. This is illustrated in Fig. 1. For a distorted wave such as that illustrated in (a) we get the oscilloscope trace shown in (b) when the phase shift of the second harmonic is  $90^\circ$ . The increase of waveform distortion with distance could be observed by watching this type of distortion on the scope. Since there is a phase shift of  $180^\circ$  in each of the harmonics when a rigid reflector is used and  $0^\circ$  when a pressure release reflector is used, the corresponding oscilloscope traces differed markedly from each other.

In Fig. 2 is given a series of pictures of oscilloscope traces which show the transducer output at increasing distances from the source. It can be seen that the distortion of the waveform increases with distance. As indicated, reflection from a solid boundary produced the waveforms shown in the second row of pictures where it can be seen that the distortion continues to

increase with increasing distance. On the other hand, the lower part of the figure shows that the wave on reflection from a pressure-release boundary is inverted; i.e. it is distorted in the "wrong direction." Therefore, the distortion decreases with increasing distance until at a distance from the pressure-release reflector almost equal to that between the transducer and the boundary, the waveform is again essentially sinusoidal. On progressing farther, the wave becomes sinusoidal and then distorts again.

This behavior led us to wonder what the situation would be if we were to use such reflectors to set up standing waves, using continuous waves instead of pulses. We investigated these standing finite amplitude waves by looking at the optical diffraction effects.

One of the reasons for wanting to make this investigation is that, although all of the details were not worked out initially, the study of standing waves by optical refraction resulted in the first indication that finite amplitude distortion in liquids was large enough to produce interesting optical effects. Now, Zonkel has solved the problem of the diffraction of light by progressive finite amplitude waves and Cook has solved the problem of the diffraction of light by a standing sinusoidal wave of arbitrary standing wave ratio. We, therefore, are able to use their results in considering the diffraction of light by standing finite amplitude waves of arbitrary standing wave ratio. This problem is rather

complicated, and has not been worked out in detail; however, having these results using the rigid and the pressure release boundary, some conclusions about the diffraction of light by standing finite amplitude waves can be made.

Consider now what would happen if we had a standing wave made up of distorted waves travelling in opposite directions, the distortion being such that the leading edges are steepest. This is the situation one finds with a rigid reflector if the standing wave system is observed near the reflector. The diffraction pattern produced by such a system of waves would be symmetrical; i.e. the positive and negative diffraction orders will be of equal intensity. This should be true for all sound intensities. On the other hand, the standing wave system found when we have a pressure release reflector would be the superposition of, say, a wave going to the right with a steep leading edge with a wave going to the left with a steep trailing edge. Such a system would produce a diffraction pattern which is asymmetrical since the distortions of the waves are in the same direction. The assumption made here about the standing wave system is strictly valid only in the vicinity of the reflector. We, therefore, passed a beam of light as close as possible to the reflector and observed how the character of the diffraction pattern changed as the distance between source and reflector was increased.

The character of the entire diffraction pattern can be realized by observing how the first diffraction order changes with

increase of sound intensity and comparing the experimental values with what would be expected if one had sinusoidal waves. This comparison was made by first setting the source and reflector within 3 cm. of each other. At this distance, whatever be the reflector, the waves are sinusoidal. Therefore, one can calculate the intensity of light in the diffraction orders from the expression

$$I_n = \sum_v J_n^2(v) J_{n-nv}^2(v/a)$$

where

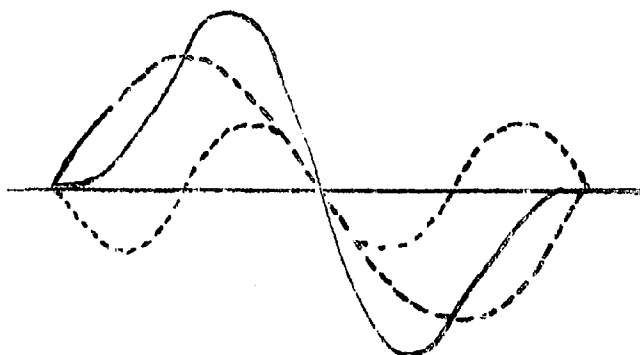
$$v = \frac{2\pi PL}{\lambda}$$

As is indicated, the quantity  $v$  is proportional to the sound pressure amplitude  $P$ , which is proportional to the quartz voltage.  $K$  is the piezo-optic coefficient and  $L$  is the width of the sound beam. The proportionality constant between the quartz voltage and  $v$  was found by observing that whatever the standing wave ratio the intensity of the zero order all pass through the common value of 30% of the incident intensity at the value  $v = 1.4$ . With this calibration it is possible to compare the theoretical values of light intensity in any order with the experimental ones. We look at the first order curves.

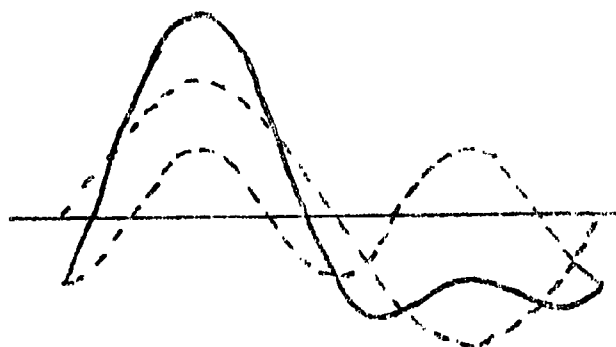
Fig. 3 shows how the light intensity in the first orders varies with increasing sound pressure when the rigid reflector is used. The positive and negative orders were of equal intensity to within experimental error. The symmetry of the diffraction pattern is thus verified. It will be noted that as the distance

is increased between source and reflector the experimental points deviate more and more from the theoretical curve. This is caused by the fact that the second harmonic is increasing. This second harmonic does not affect the symmetry of the pattern, but it does affect the intensity. This, then, is not cancellation of the type Zankel and Mayer produced when they used two progressive finite amplitude waves travelling in the same direction.

On the other hand, the presence of second harmonic can produce asymmetry. This is shown in Fig. 4 where we have used a pressure release reflector. At 3 cm between source and reflector the second harmonic is not present even for the highest amplitude used. As the distance increases the second harmonic increases and this time produces an asymmetry of the pattern. This asymmetry is a measure of the amount of the second harmonic present. It can thus lead to a determination of the waveform in space. (The time dependence of the wave form can be obtained from a study of the way the diffraction pattern is modulated by using an ac photomultiplier. Thus, the presence of the least stable waveform makes it possible to use optical methods to determine the waveform distortion of a standing finite amplitude wave.



INCIDENT DISTORTED WAVE



RESULTING CRO TRACE

FIG. 1  
EFFECT OF PHASE SHIFT OF SECOND  
HARMONIC IN RESONANT TRANSDUCER

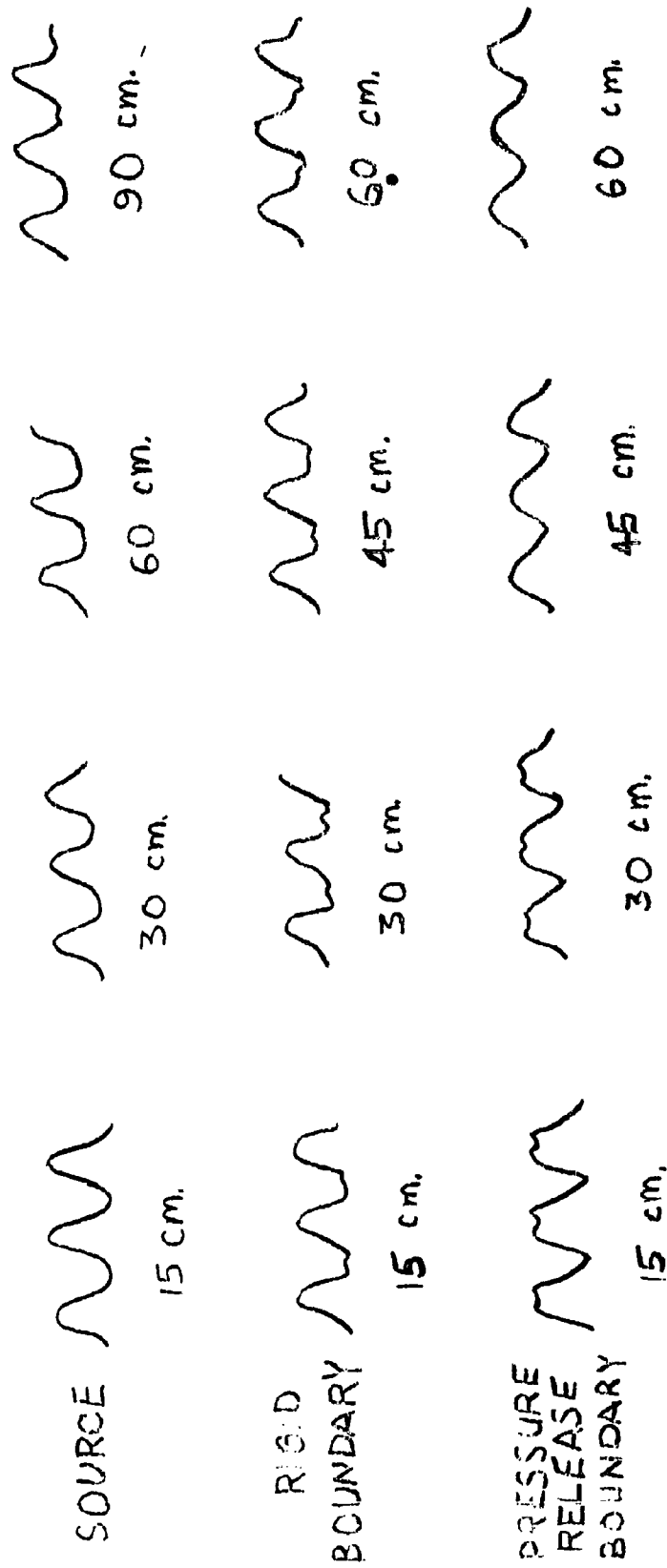


FIG. 2.  
REFLECTION OF FINITE AMPLITUDE WAVE

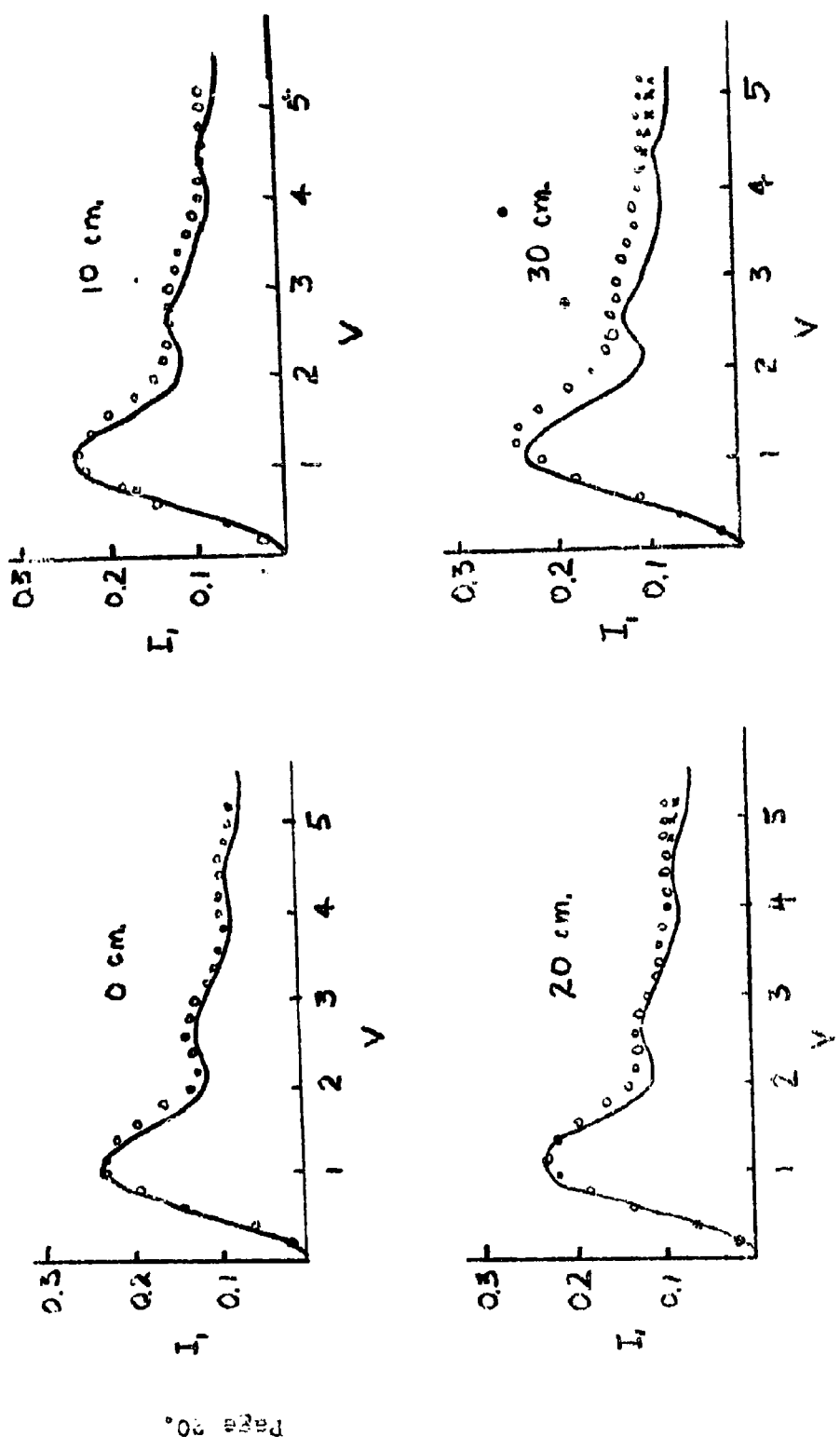


FIG. 3

— STANDING WAVE RATIO 1:1 RIGID REFLECTOR

○ ○ ○ ○ FIRST ORDER

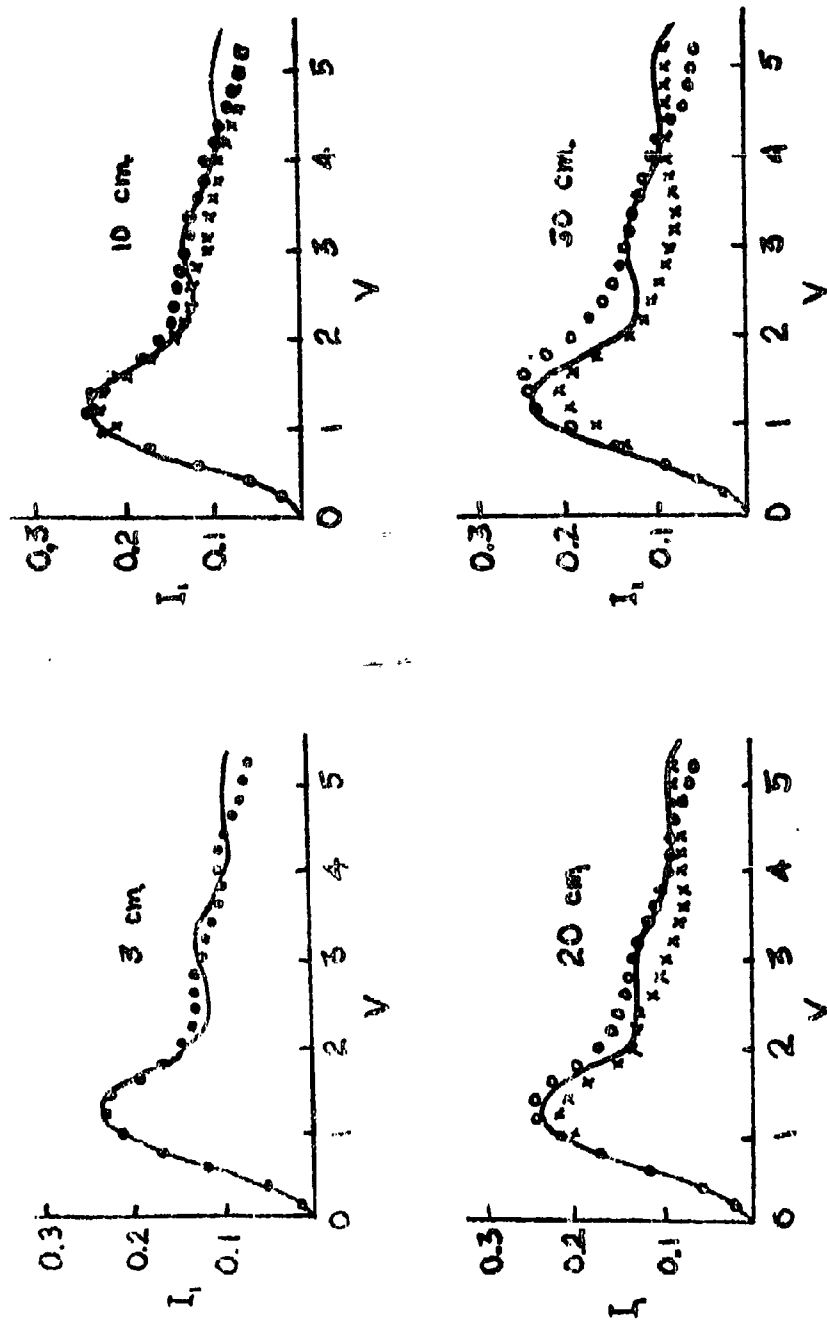


FIG 4

— STANDING WAVE RATIO 5:4    - - - - - FIRST ORDER  
PRESSURE RELEASE BOUNDARY

# DISTRIBUTION LIST FOR UNCLASSIFIED TECHNICAL REPORTS

Office of Naval Research (Code 411)  
Department of Navy  
Washington 25, D. C. (2 copies)

Director  
U. S. Naval Research Laboratory  
Technical Information Division  
Washington 25, D. C. (6 copies)

Director  
U. S. Naval Research Laboratory  
Sound Division  
Washington 25, D. C. (1 copy)

Commanding Officer  
Office of Naval Research Branch Office  
1030 East Green Street  
Pasadena 1, California (1 copy)

Commanding Officer  
Office of Naval Research Branch Office (1 copy)  
The John Crerar Library Building  
86 East Randolph Street  
Chicago 1, Illinois (1 copy)

Commanding Officer  
Office of Naval Research Branch Office  
Box 39, Navy No. 100  
FPO, New York (5 copies)

Office of Technical Services  
Department of Commerce  
Washington, D. C. (1 copy)

Armed Services Technical Information  
Agency  
Arlington Hall Station  
Arlington 12, Virginia (10 copies)

Commander  
U. S. Naval Ordnance Laboratory  
Acoustics Division  
White Oak  
Silver Spring, Maryland (1 copy)

Commanding Officer and Director  
U. S. Navy Electronics Laboratory  
San Diego 52, California (1 copy)

Director  
U. S. Navy Underwater Sound  
Reference Laboratory  
Office of Naval Research  
P. O. Box 3629  
Orlando, Florida (1 copy)

Commanding Officer and  
Director  
U. S. Navy Underwater Sound  
Laboratory  
Fort Trumbull  
New London, Connecticut  
(1 copy).

Commander  
U. S. Naval Air Development  
Center  
Johnsville, Pennsylvania  
(1 copy)

Director  
National Bureau of Standards  
Connecticut Avenue and  
Van Ness St. N.W.  
Washington 25, D. C.  
(Attn: Chief of Sound  
Section) (1 copy)

Office of Chief Signal  
Officer  
Department of the Army  
Pentagon Building  
Washington 25, D. C. (1 copy)

Commanding Officer and  
Director  
David Taylor Model Basin  
Washington 7, D. C. (1 copy)

Superintendent  
U. S. Navy Postgraduate  
School  
Monterey, California  
(Attn: Prof. L. E. Kinsler)  
(1 copy)

Chesapeake Instrument Corporation  
Shadyside, Maryland (1 copy)

National Science Foundation  
1520 H Street N. W.  
Washington, D. C. (1 copy)

Commanding Officer  
U. S. Navy Mine Defense Laboratory  
Panama City, Florida (1 copy)

U. S. Naval Academy  
Annapolis, Maryland  
(Attn: Library) (1 copy)

Harvard University  
Acoustics Laboratory  
Division of Applied Science  
Cambridge 38, Mass. (1 copy)

Brown University  
Department of Physics  
Providence 12, R. I. (1 copy)

Western Reserve University  
Department of Chemistry  
Cleveland, Ohio  
(Attn: Dr. E. Yeager) (1 copy)

University of California  
Department of Physics  
Los Angeles, California (1 copy)

University of California  
Marine Physical Laboratory of the  
Scripps Institution of  
Oceanography  
San Diego 52, California (1 copy)

Bell Telephone Laboratories  
Whippany, N. J. (1 copy)

Director  
Columbia University  
Hudson Laboratories  
145 Palisades Street  
Dobbs Ferry, N. Y. (1 copy)

Woods Hole Oceanographic Institute  
Woods Hole, Massachusetts (1 copy)

University of Michigan  
Engineering Research Institute  
Ann Arbor, Michigan  
(Attn: Dr. J. C. Johnson)  
(1 copy)

Dr. J. R. Smithson  
Electrical Engineering Department  
U. S. Naval Academy  
Annapolis, Maryland (1 copy)

Laboratory of Marine Physics  
Yale University  
Box 1916 Yale Station  
New Haven 11, Conn. (1 copy)

Lamont Geological Observatory  
Columbia University  
Torrey Cliffs  
Palisades, N. Y. (1 copy)

The Catholic University of America  
Department of Physics  
Washington, D. C. (1 copy)

Massachusetts Institute of  
Technology  
Laboratory of Electronics  
Cambridge 39, Mass.  
(Attn: Dr. U. Ingard) (1 copy)

Director  
Ordnance Research Laboratory  
Pennsylvania State University  
University Park, Pa. (1 copy)

Defense Research Laboratory  
University of Texas  
Austin, Texas (1 copy)

Bureau of Ships (Code 845)  
Department of the Navy  
Washington 25, D. C. (1 copy)

Bureau of Aeronautics (W-43)  
Department of the Navy  
Washington 25, D. C. (1 copy)

Bureau of Ordnance (ReUlc)  
Department of the Navy  
Washington 25, D. C. (1 copy)

U. S. Navy SOPAR Station  
APO #856, c/o Postmaster  
New York, New York  
(Attn: Mr. G. R. Hamilton)  
(1 copy)

John Carroll University  
University Heights  
Cleveland 18, Ohio  
(Attn: E. F. Carome) (1 copy)

Edo Corporation  
College Point, L. I., N. Y.  
(Attn: c. Loda) (1 copy)

Mr. Fred O. Briggson  
ONR Resident Representative  
University of Michigan  
820 East Washington St.  
Ann Arbor, Michigan (1 copy)

Naval Ordnance Test Station  
Pasadena Annex  
3203 E. Foothill Blvd.  
(Pasadena 8, California (1 copy)

Applied Physics Laboratory  
University of Washington  
Seattle, Washington (1 copy)

Dr. W. J. Fry  
Biophysical Research Laboratory  
University of Illinois  
Urbana, Illinois (1 copy)

UNCLASSIFIED

UNCLASSIFIED